Int. J. Heat Mass Transfer. Vol. 15, pp. 555–557. Pergamon Press 1972. Printed in Great Britain

SHORTER COMMUNICATIONS

REMARKS ABOUT 'GENERAL SOLUTION OF THE EQUATIONS OF MULTICHANNEL HEAT EXCHANGERS' [1]

A. SETTARI

Dept. of Mechanical Engineering, University of Calgary, Alberta, Canada

(*Received* 17 *December* 1970)

NOMENCLATURE

- n, number of channels;
- t_i, temperature of fluid in channel i;
- $\boldsymbol{t}$, temperature vector, $\boldsymbol{t} = (t_1, t_2, \ldots, t_n)^T$;
- x, space coordinate along the stream;
- L, length of channel in x-direction;
- k_{ik}, overall conductance for heat transfer between channels i and k;
- k_e, effective heat transfer conductance;
- h_{ik}, common perimeter of channels i and k;
- W, fluid capacity rate (water equivalent), $W = Qc$;
- Q, flow rate of fluid;
- c, specific heat of fluid;
- a_{ik}, specific heat transference from channel i to channel k, $a_{ik} = \dfrac{k_{ik} h_{ik}}{W_i}$;
- F, area for heat exchange, $F = L\,h$;
- $\bar{a}_{ik}$, dimensionless heat transfer conductance, $\bar{a}_{ik} = \dfrac{k_{ik} F_{ik}}{W_i}$;
- A, coefficient matrix of the resulting system of differential equations;
- I, unit matrix;
- $\boldsymbol{K}_i$ eigenvectors of matrix A, $\boldsymbol{K}_i = (K_{1i}, K_{2i}, \ldots, K_{ni})^T$;

Greek symbols

- λ_i, eigenvalues of matrix A;
- ξ, η, dimensionless heat transfer conductances of two streams in Fig. 1;
- ϕ, dimensionless space coordinate, $\phi = x/L$;

THE SYSTEM of equations for parallel-flow multichannel heat exchanger using the linear formulation is, in the notation of [1]

$$\frac{dt_i}{dx} + \sum_{k=1}^{n} a_{ik}(t_i - t_k) = 0 \qquad i = 1, 2, \ldots, n \quad x \in \langle 0, L) \tag{1}$$

where $a_{ik} = k_{ik}\,h_{ik}/W_k$ is the specific heat transference (number of transfer units per unit of coordinate x). Several authors have investigated the solution of system (1) for specific cases and Wolf [1] presents a general procedure for obtaining exact solutions. System (1) can be written in compact matrix form as:

$$A\boldsymbol{t} = \frac{d\boldsymbol{t}}{dx} \tag{2}$$

where A is the $n \times n$ matrix,

$$A = \begin{bmatrix} -\sum\limits_{k=1}^{n} a_{1k} & a_{12} \ldots\ldots\ldots\ldots & a_{1n} \\ a_{21} & -\sum\limits_{k=1}^{n} a_{2k} \quad a_{23} \ldots\ldots & a_{2n} \\ \vdots & & \\ a_{n1} & & -\sum\limits_{k=1}^{n} a_{nk} \end{bmatrix}$$

If all the eigenvalues of matrix A are distinct, the general solution of (2) is given by

$$\boldsymbol{t} = \sum_{i=1}^{n} \boldsymbol{K}_i\, e^{-\lambda_i x} \tag{3}$$

where λ_i and K_i are eigenvalues and eigenvectors of matrix A. If, however, some of the roots are repeated, the problem of finding the general solution becomes more complicated (see, for example, [2] or [3]).

Wolf proves that all eigenvalues are distinct; however, his proof contains an invalid assumption that multiple zero roots cannot exist. Moreover, he does not prove that the eigenvalues are real, which is of theoretical as well as of practical importance. We will here demonstrate the answer and show that Wolf's assumption is invalid.

(1) *All eigenvalues of matrix* A *are real*

Proof: The eigenvalues of matrix A are a solution of the characteristic equation $|A - I\lambda| = 0$.

Premultiplying this equation by the diagonal matrix B,

$$B = \begin{bmatrix} W_1 & & & & & & \\ & W_2 & & & & & \\ & & W_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & W_n \end{bmatrix} \qquad |B| \neq 0$$

results in the eigenvalue problem,

$$|C - B\lambda| = 0$$

where $C = BA$. As $|BA| = |B||A|$ for any matrix then

$$|C - B\lambda| = |B(A - I\lambda)| = |B||(A - I\lambda)| = 0$$

so that all eigenvalues of matrix A and the above equation are identical.

Now; (a) matrix C is symmetric as

$$C = \begin{bmatrix} -\sum_k k_{1k}h_{1k} & k_{12}h_{12} \ldots k_{1n}h_{1n} \\ k_{21}h_{21} & -\sum_k k_{2k}h_{2k} \ldots\ldots k_{2n}h_{2n} \\ \vdots & \\ k_{n1}h_{n1} \cdots\cdots\cdots\cdots & -\sum_k k_{nk}h_{nk} \end{bmatrix}$$

and $k_{ji}h_{ji} = k_{ij}h_{ij}$ by definition.

(b) matrix C is diagonally dominant with diagonal entries of the same sign; this implies together with (a) that C is negative semidefinite ([5], p. 24)

(c) B is symmetric, nonsingular.

Then all finite eigenvalues of A are real as follows from [4], Theorem 8.6.

The physical meaning of this is obvious; the solution does not possess any oscillatory component.

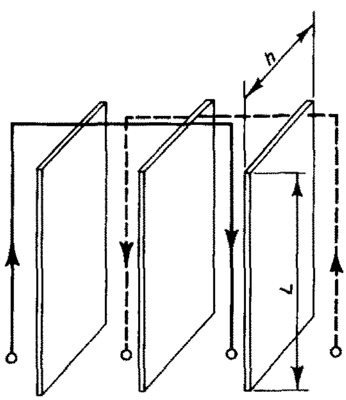

FIG. 1.

(2) *All nonzero eigenvalues are distinct; however, zero multiple eigenvalues may exist.*

The first part has been proved by Wolf [1], if his assumption (page 905), that multiple zero roots cannot exist is omitted from the consideration. In order to prove this contradiction, consider the following simple example (cf. [6]).

Let the geometry of the exchanger be that given in Fig. 1. Assuming k = constant and introducing the dimensionless constants

$$\xi = \frac{kF}{W_1} = -\frac{kF}{W_3} = a_{12}L = a_{32}L.$$

$$\eta = \frac{\xi}{u} = -\frac{kF}{W_2} = \frac{kF}{W_4} = a_{21}L = a_{23}L, \text{ where } F = Lh$$

and the dimensionless variable $\phi = \frac{x}{L}$, yields

$$At = \frac{dt}{d\phi}$$

where

$$A = \begin{bmatrix} -\xi & \xi & 0 & 0 \\ -\frac{\xi}{u} & \frac{2\xi}{u} & -\frac{\xi}{u} & 0 \\ 0 & -\xi & 2\xi & -\xi \\ 0 & 0 & \frac{\xi}{u} & -\frac{\xi}{u} \end{bmatrix}$$

The eigenvalues of matrix A are the solution of the equation

$$\lambda^2\left[\lambda^2 - \lambda\xi\left(\frac{1}{u} + 1\right) - 2\xi^2\left(\frac{1}{u^2} - \frac{1}{u} + 1\right)\right] = 0$$

which gives the multiple root $\lambda_1 = \lambda_2 = 0$

Similarly changing direction of one flow, for example, the signs of W_2 and W_4 results in the characteristic equation

$$\lambda\left[\lambda^3 + \lambda^2\xi\left(\frac{1}{u} - 1\right) - 2\lambda\xi^2\left(1 + \frac{1}{u^2}\right) + 2u\xi^3\left(1 - \frac{1}{u}\right)\right] = 0$$

which again results, for $u = 1$, in a multiple zero root.

Consequently, Cramer's rule cannot be used to obtain the eigenvectors, corresponding to the multiple root λ_m, as in this case, the rank of matrix $(A - \lambda_m 1)$ may be $n - 2$ (see Hildebrand [7]), and all minors of order $n - 1$ are zero. Thus equations (7)–(10) in [1] do not serve the general solution, if the zero root is really multiple.

REFERENCES

1. J. WOLF, General solution of the equations of parallel-flow multichannel heat exchanger, *Int. J. Heat Mass Transfer* 7, 901–919 (1964).
2. E. A. CODDINGTON and N. LEVINSON, *Theory of Ordinary Differential Equations*. McGraw-Hill, New York (1955).
3. R. A. FRAZER, W. J. DUNCAN and A. R. COLLAR.

Elementary Matrices. Cambridge University Press. London (1960).
4. P. LANCASTER, *Lambda-Matrices and Vibrating Systems*, Pergamon Press, Oxford (1966).
5. R. S. VARGA, *Matrix Iterative Analysis*. Prentice Hall, New Jersey (1962).
6. U. MENNICKE, Wärmetechnische Eigenschaften der verschiedenen Schaltungen von Plattenwärmeaustauschern, *Kältetechnik* **11**, 162–167 (1959).
7. F. B. HILDEBRAND, *Methods of Applied Mathematics*, second ed. Prentice-Hall, New Jersey (1965).

Int. J. Heat Mass Transfer. Vol. 15, pp. 557–558. Pergamon Press 1972. Printed in Great Britain

A SMOKE GENERATOR FOR USE IN FLUID FLOW VISUALIZATION

J. P. YU, E. M. SPARROW and E. R. G. ECKERT
Department of Mechanical Engineering, University of Minnesota, Minneapolis, Minnesota, U.S.A.

(*Received* 15 *July* 1971)

INTRODUCTION

FLOW visualization can be used to provide decisive insights into the processes which occur in many fluid mechanics and convective heat-transfer problems. In gases, flow visualization is usually accomplished by the injection of smoke. Various ways of producing such smoke have been reported, among which are chemical reactions and burning of combustible materials such as cigars. Some of the methods are of rather limited applicability because they produce smoke which is either corrosive, flammable or toxic. Smoke generators for short-time flow visualization are not difficult to devise, especially for the low rates of smoke utilization appropriate to laminar conditions. On the other hand, long-time visualization of highly turbulent flows requires that a high rate of smoke production be sustained for an extended period of time. This note describes a general purpose smoke generation system that fulfills these requirements while preserving simplicity of construction and operation. In addition, representative flow field photographs of a turbulent, rotating airflow in a cylindrical enclosure will be presented to demonstrate the effectiveness of the smoke generator.

SMOKE GENERATION SYSTEM

A schematic diagram of the smoke generation system is shown in Fig. 1. The smoke is produced by the burning of wood shavings in a steel pipe. As is seen in the figure, compressed air is ducted to the pipe inlet and, in passing through the pipe, participates in the combustion process. The thus-produced smoke flows through copper tubing to the primary filter, which consists of a steel pipe packed with steel wool. The primary filter is immersed in a water bath to reduce the temperature of the smoke stream. The smoke is then conveyed by plastic tubing to a second filter, which is a glass bottle (to facilitate viewing of the smoke) packed with steel wool. The second filter is provided with two exit ports, one of which leads to the fluid flow test facility while the other of which is a by-pass for venting the smoke.

In the present experiments, the smoke was ducted into a manifold, from which it was distributed to as many as 24 ($\frac{3}{16}$ in. dia.) injection holes installed in the walls of the test apparatus. With all 24 holes in use, the smoke generation system provided sufficient smoke for about 25 min of continuous visualization of the flow.

The wood shavings used in the smoke generator were produced with a standard wood-shop planer. White pine was found to be a suitable wood. To load the smoke generator with a charge of shavings, the steel pipe was decoupled from the system. The shavings were introduced from both ends of the pipe and compacted with the aid of a rod. The fully charged pipe was then screwed to pipe reducers at its upstream and downstream ends.

The upstream pipe reducer was fitted with an electrical resistance heating element whose function was to ignite the wood shavings. In order to ensure ignition, shavings were also tightly packed into the upstream pipe reducer. The heating element was coiled nichrome wire, a coil being used to permit stretching and displacement of the element when the shavings were being loaded. Power was supplied to the heating element through a pair of copper connectors